COLAUMS: Women in Maths Day Edition

Introduction

Hi everyone, welcome to the special edition of ColAUMS, a celebration of International Women in Mathematics Day. In this edition we will treat you with an article on Maryna Viazovska, one of only two female Fields Medalists. We have an interview with Senior Lecturer Dr Elizabeth Jensen Young, an applied mathematician working on some math out of this world! We also have a letter from the president of AUMS, Makon Westdorp, talking about our new editorial process. Please, enjoy this dose of math, and have a mathematical day!

The Sphere Packing Problem

Mathematics can be pretty cursed. In fact, look no further than the (current) most optimal way to pack 17 squares in a larger square.

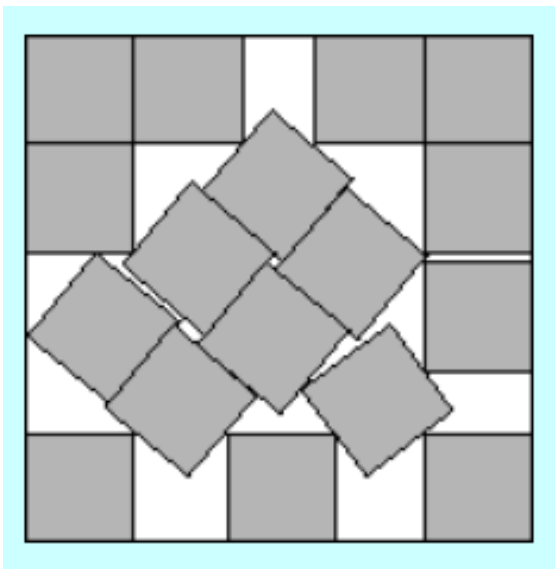


Figure 1: The optimal way to pack 17 squares. (Erich Friedman)

Packing problems, in general, are a set of optimisation mathematical problems revolving around the idea of packing objects inside containers whose dimensions are minimised.

Another unique feature of mathematics is how we can turn real world concepts into abstracted theories. While packing obviously have real world applications, the problems that we will explore today are in dimensions the human mind cannot even imagine.

Meet Maryna Viazovska, a Ukrainian mathematician, and the second woman ever to win the prestigious Fields Medal. Viazovska solved the problem of sphere packing in 8 dimensions, in infinitely extending 8D space.

Figure 2: Maryna Viazovska with her fields medal in 2022.

Viazovska's research involved finding the optimal lattice structure of hyperspheres in 8 dimensions. A *hypersphere* is the set of all points equidistant from a given centre point.

As opposed to packing a finite number of hyperspheres, the aim of Viazovska's research was to find an optimal repeating structure, and thus the notion of minimizing the dimensions of some container no longer makes sense. Instead, we define a property called *density*, which refers to the fraction of space taken up by the objects that are being packed.

It may be worth thinking about this problem in lower dimensions first. For example, in 2D, the best circle packing arrangement is in a hexagonal structure.

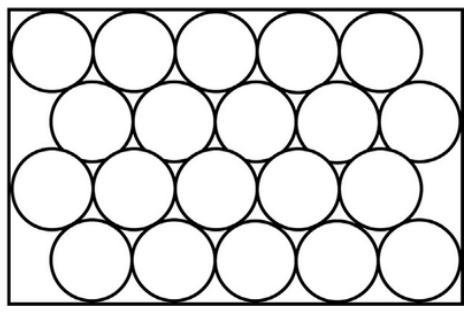


Figure 3: Optimal Sphere Packing Lattice in 2D

In 3D, this would look like layers of these hexagonal structures. Both of these structures have been proven optimal.

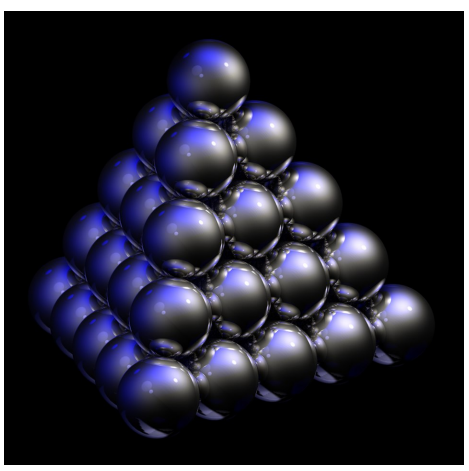


Figure 4: Optimal Sphere Packing Lattice in 3D

Viazovska proved that in 8 dimensions, the optimal lattice structure is the E8 lattice structure.

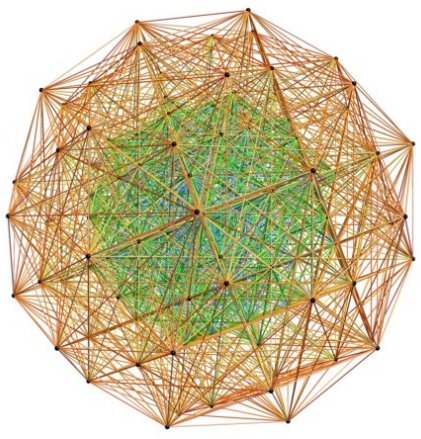


Figure 5: The E8 lattice as a graph.

Rather than trying to visualise what the structure looks like, the graph shows the neighboring hyperspheres of a particular hypersphere, of which there are 240. The vertices represent which of those hyperspheres touch. This structure has a density of $\frac{\pi^4}{384} \approx 0.25$.

Despite such a complex structure, the symmetries of the E8 lattice allowed for a remarkably easy proof - a unique characteristic of the 8th dimension that allows it to be the only dimension (at the time) to have a solved structure, other than 1, 2 and 3 dimensions.

Maryna Viazovska's solution to the sphere packing problem in 8 dimensions, and her subsequent Fields Medal is a symbol of progression for women in a field that has historically underrepresented women. We hope that her hard work and dedication can inspire you to pursue your own challenges in mathematics.

An Interview with Dr Elizabeth!

Meet Dr Elizabeth Jensen Young, our beloved Differential Equations and Maths 1M Lecturer! In this edition, we had the chance to interview Elizabeth. Elizabeth is a wonderful mathematician working at the University of Adelaide, and it was a joy to meet and discuss some mathematics and opinions with her. I wish to extend my thanks yet again for this amazing interview. Thank you again Elizabeth.

- Makon Westdorp

Figure 6: Dr Elizabeth Jensen Young

Did you always want to work in this field? When did you know you wanted to work in STEM?

So, the bigger broader picture of STEM I knew at a younger age, but being a University lecturer? No, I didn't have that ambition as a younger child. When I was 10 years old, I wanted to make a go-kart! I went to the local university that my parents worked at, and was like "I need your equipment, I need you to teach me how to weld, and solder some electronics, and how an engine works and all these other things." The university was more than accommodating, and it was awesome. I knew I wanted STEM, and I think my beginning love was physically building stuff.

What is your experience like as a woman in STEM?

So, I went to an all-women's university in the United States for undergrad. In high school, I was one of two women taking high level physics, and in university, I was one of, say, eight majoring in physics (all women), and in graduate school, I was one out of the two women in my cohort of about 15 students starting their PhDs in Mechanical and Aerospace Engineering that year. Currently, get questioned about once a month about being a women in STEM. In graduate school I asked my advisor if his kids were majoring in STEM. His response was "No, I have daughters." He could have just said "No, they have other interests." Gender stereotyping becomes obvious as a woman in STEM, there is just a societal Norm that women can't do math, and it just wears your confidence down. Actually interviewing in this math department, I was told "Oh, coming from physics and engineering, you'll actually be happy how many women are in maths.". I think people are becoming much more aware that this cultural norm of "girls are bad at math" is not okay. It's amazing how young it starts, that when writing numbers on a whiteboard in primary school, one person can do it better because of their gender? It doesn't make sense. I think that part is changing, at least the awareness.

Do you have anything to say for women trying to get into a field like mathematics?

One thing that I was surprised by, getting into math and science was everyone thinks that I am a "geek". They think my interest in math means I have preferences over Marvel, or DC. Star Trek, or Star Wars. I need to have strong opinions about all these cultural things related to science and math. My words of wisdom are "You don't need to fit the mould! Yes, a lot of people that like science fiction go into science, but you don't need to fit yourself into that box! Be yourself!" I get all the time that people are surprised that I am not a geek, I don't have 200 digits of pi memorised. You do not need to fit the mould and the stereotype. I am in math because I like math, not my opinions on science fiction! If you like math, keep going! Don't get locked into one field during undergrad, that's what your PhD is for. You won't know what this other field is, until you try it. A lot of experiences in your 20's should be things that don't work, so you understand when things do work!

What is your research? What are your interests, and what do you work on?

My research is looking for light from planets in solar systems that aren't our own. It helps us answer the question of "What do solar systems normally look like?" Up until a few decades ago, we had a sample size of 1, our Solar System. Statistically, that's a terrible sample size, and it is natural to collect more data to understand what is common and what is unique.

I design optics to go onto telescopes and write algorithms to take images of a solar system and filter out the light from the star to find planets. I try to be creative with my optics to distort the image, so we have dark spots around the star. Then when a planet is within this dark spot, we can see it with our telescope! One of the main pieces of math I use for this is the Fourier Transform, to look between the pupil plane (similar to where the telescope first begins collecting light) and the camera, the image plane. To create the dark spots on either side of the star I was talking about, I put sine waves on a deformable mirror in the pupil plane. The frequency of those waves are related to the position of the spots. I can therefore design optics and move the mirror in the pupil plane based on what I want to see in the image plane Now I can use this to control where the dark spots are and use that to find planets.

I use the biggest telescopes in the world, the 8-10m class ones. I go into a cleanroom lab and use a process similar to how computer chips are made to make telescope optics, usually about an inch big.

This field in astronomy is extremely new, as I believe the first exoplanet was first imaged in 2004. I really liked that newness as a student, because it helped me feel like my ideas counted, because no one knew what they where doing. I use the tools developed from white dudes 200 years ago, but making relevant and new discoveries.

The New Editorial Process

Dear reader,

Earlier this year we released a retraction notice for our previous edition of ColAUMS. While I do not wish to dwell on the details, it prompted some very valuable conversations within AUMS and the School of Computer and Mathematics Science. As a result of these discussions I am pleased to share that we have introduced a new editorial process.

This is an opportunity to improve. Our goal has always been to make ColAUMS memorable, but not like last edition. I want to show the creativity and beauty of mathematics, and I believe this process will help us do that.

Heres how it works:

Contributions
Each contributor submits their piece along with a signed Integrity Statement. This is a short document that acts as both a personal declaration of authenticity, and a reminder of our responsibilities under the student code of conduct. This statement confirms that all the work is original, or properly referenced, and AI-free. We as a club believe that if someone did not put in the effort to write a piece, no one should put in the effort to read it. This integrity statement also requires contributors to provide proof that this was not plagiarised, by showing a TurnItIn receipt,aswell as proof that non-public figures have consented for the use of their name and words, such as for interviews with staff. Public figures are exempt from this requirement.

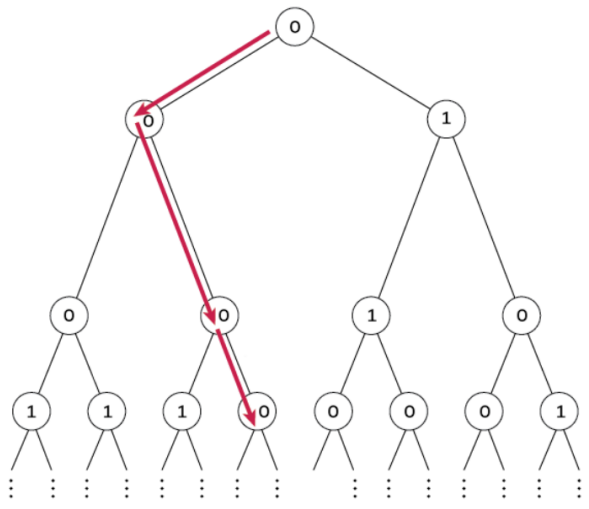
Lead Editor Review
All the work from each individual contributor is then collated by the Lead Editor into the beautiful ColAUMS edition you are reading now. This lead editor ensures all integrity statements are complete and signed correctly.

Final Check and Feedback
Before publication, the edition is reviewed by an independent member of the committee - A student that has not worked on the current edition of ColAUMS. This independent review checks all the documents again, and provides some crucial feedback before publication. I believe this process shows the right balance. It has enough rigor to protect the publication and the contributors, but light enough to still make it feasible to contribute.

If you are thinking about contributing to ColAUMS, now is the time! Whether you have an idea for an article, a puzzle or an interview, we would love to hear from you. With this new process in place, we are excited to keep building ColAUMS's future.
Warm regards,
- Makon Westdorp
- President

Puzzle

This edition's puzzle is adapted from Jane Street's April puzzle.



Given an infinite, complete binary tree, with nodes labelled 0 with probability 0.75, and 1 with probability 0.25, what is the probability that there exists an infinite path down the tree where the nodes sum to a maximum of 1?

Submit your answers to the AUMS Discord server!

https://discord.gg/kVrr9jAv6N

[0]https://en.wikipedia.org/wiki/Packing_problems
[0]https://magazine.paperhive.org/viazovska-sphere-packing-problem/
[0]https://www.popsci.com/science/fields-medal-winners-mathematicians/
[0]https://impa.br/notices/how-maryna-viazovska-cracked-the-higher-dimension-sphere-packing-code/
[0]https://mathvoices.ams.org/featurecolumn/2022/09/01/eight-dimensional-spheres-and-the-exceptional-e_8/