ColAUMS Space

NEWSLETTER OF THE ADELAIDE UNI MATHS SOCIETY | ISSUE 1, 2023

In this issue

Editor's Welcome
The Origins of Chaos
Why we Study Category Theory
Christine O'Keefe
Sam and the Aliens
Puzzle 2
Comic 2

Contact details

AUMS email
mathsatadelaide@gmail.com
AUMS website
aums.org.au
AUMS Facebook page
facebook.com/UofAMathsSociety
AUMS Instagram
@adelaideunimaths
AUMS Discord
discord.aums.org.au
Want to contribute? Contact the editor at
colaums.space@gmail.com

PASS

Mathematics in university can be very challenging but you don't have to do it alone. In PASS, the leaders can lead you through confusing questions or concepts. They are there to create this study mindset from a student's point of view who has been there and successfully completed it. It is emphasised to students that we cannot breach academic integrity but you can learn to have discussions with your peers to deepen your knowledge of concepts in a safe environment. All you need to do is attend the PASS and tap into this weekly resource available to you as per https://www.adelaide.edu.au/pass/course-and-session-times

- Thoa (Twa), Maths 1M PASS leader

Editor's Welcome

Bad news for column fans this edition as we welcome you to the *Space* special edition of ColAUMS Space. In this edition read about: The cosmological orgins of chaos; An infinite earth and category theory; A University of Adelaide Professor who worked in Projective *Space*; A short story about aliens; and a comic.

But first, a quick addendum with regards to Issue 0. In the previous issue we mentioned an incident in which Horace Lamb responded to a newspaper article. We have since found the original article on Trove.[1] It is certainly an interesting read.

- George Savvoudis, ColAUMS Editor

The Origins of Chaos

Consider the following problem: Three planets move through space under only the force of gravity. Given their initial conditions, predict the motion of the planets.

Intuitively, this feels like a problem which should be solvable, in that, if we know the masses of the planets and the distances between them we can calculate the forces instantaneously. Hence we set up the following system.

Let $r_i = (x_i, y_i, x_i)$ be the position of planet i. Likewise let m_i be the mass of planet i. If G is the gravitational constant, then

$$\frac{d^2r_1}{dt^2} = -Gm_2\frac{r_1 - r_2}{|r_1 - r_2|^3} - Gm_3\frac{r_1 - r_3}{|r_1 - r_3|^3}$$
$$\frac{d^2r_2}{dt^2} = -Gm_1\frac{r_2 - r_1}{|r_2 - r_1|^3} - Gm_3\frac{r_2 - r_3}{|r_2 - r_3|^3}$$
$$\frac{d^2r_3}{dt^2} = -Gm_1\frac{r_3 - r_1}{|r_3 - r_1|^3} - Gm_2\frac{r_3 - r_2}{|r_3 - r_2|^3}$$

In 1912, Karl Fritiof Sundman gave an analytic solution to this system in the form of a power series which converges for most practical initial conditions. However, this solution has been criticised for providing no qualitative information and that the rate of convergence is too slow to be of any practical use.

The solution by Sundman, however, is the closest we have come to a general solution.

To understand the difficulties of this problem it is useful to consider a related problem, the 'restricted' three body problem.

The set up is similar, three bodies acting under gravitational influence, however, we assume one body is of insignificant mass, and so does not affect the trajectories of the two more massive objects. This problem is not unrealistic, and can be used to describe the sun-earth-moon system, or, say, an earth-moon-saterlite system.

This is not to say the restricted three body problem is easy. It is remarked that Newton said of it '... his head never ached but with studies on the moon.' Euler in 1772 was the first to formally state the problem, however, it wasn't until the 1890's that Henri Poincaré gave a treatment of the problem which gave tremendous insight towards our understanding of Chaos.

Poincaré analysed the periodic solutions of the system using a very qualitative and global approach, and allowed for the discovery of asymptotic solutions and homoclinic points. In short that meant that solutions to this system:

- were sensitive to initial conditions, in that slight alterations often resulted in wildly different long term behaviour;
- had periodic solutions, of any given period;
- and had uncountably many aperiodic solutions.

These three qualitative properties definitionally describe a chaotic system.

In it's time, Poincaré's work was highly lauded for its global and qualitative approach. However, it wouldn't be until Lorenz's work in the 60's that the study of Chaos itself was properly realised as important in the research of dynamical systems.

It is remarkable to think that any person in the course of human history need only look up at the movements of the night sky and see in it concepts at the forefront of modern mathematical research.

As a reference for the above, see [2].

- George Savvoudis, ColAUMS Editor

Why We Study Category Theory

Modern mathematics often comprises the study of an object or a collection of objects with some 'structure' attached to them. Such objects do have some real-world applications however, we primarily study them for their applications in other fields of mathematics. Most of the time, these objects are themselves collections called 'sets'. Then, their 'structure' is how the elements of the set are related to and interact with each other.

For example, we may have a set of two elements called Bruce and Lee. However, they live on opposite sides of an infinite Earth. Now, I know what you're thinking, 'Oh just your everyday infinite Earth,' but bear with me here. Essentially, we are saying that Bruce and Lee are so far apart that they will never meet or interact with each other in their lifetime (which we would also want to be infinite—just so they don't disappear on us while we are studying them).

The object defined by this set {Bruce, Lee} has pretty much no structure. It is the 'squishiest' object possible. If we had another object defined by the set {Jackie, Chan}, then we would not be able to tell this object apart from the other from afar. So, both objects would behave in 'essentially the same' (isomorphic) way.

We could equip the set {Bruce, Lee} with some additional structure to make it more 'rigid'. For example, we can define the notion of 'closeness' of elements (topology). We were already thinking of the property of elements not interacting as them being very far apart. So, why not equip Bruce and Lee with the (discrete) topology that makes them isolated? As you would expect, the topology does not make {Bruce, Lee} much more rigid. The only contribution it makes is that it allows us to compare it with other topological objects. This is not the only topology though. We can define the (trivial) topology on {Jackie, Chan} this time to make Jackie and Chan close.

Topology is not the only structure either. Instead, we can define an operation, say hyphenation, on elements. Then, we could add all new elements generated by hyphenation (such as Bruce-Lee-Lee or Lee-Bruce-Lee-Bruce) to the set. This forms a (free) semigroup. We could also add an element called 'Nothing', such that hyphenating with Nothing doesn't change the element, to form a (free) monoid. Then, we can add all inverses of elements, such that if we had an element Donnie with inverse einnoD, then Donnie-einnoD = einnoD-Donnie = Nothing, to form a (free) group. Sets equipped with an operation arise all the time in mathematics (such as numbers and addition).

Usually, we are interested in which objects are 'related' to each other and how exactly they are related as that provides us insight into the 'type' of objects they are. We do this by considering structure-preserving functions between the objects. These would be functions that map 'close' elements to 'close' elements in the case of topology and functions such that applying the operation before or after the function would not change anything in the case of semigroups. The collection of objects with similar structure together with their respective functions form a category. The study of such categories is called category theory. Fundamentally, we study category theory because it provides a universal language to describe mathematical concepts from a bird's-eye view.

- Violet Evergarden, Assistant Editor

Christine O'Keefe

In honour of Women's History Month last month, we take a look into the very recent history of one of the University's former professors: Christine O'Keefe.

The story of how Christine O'Keefe came to study Mathematics at the University of Adelaide is amusing. O'Keefe had been interested in the problem solving aspects of mathematics at school; however, she had initially enrolled to study Medicine at university. In her own words:[3]

> It was only in the last couple of weeks before university started that I changed my mind and decided to do maths instead. My elder sister was doing medicine, and that's actually part of the reason that I changed. I watched her just reading enormous text books trying to remember as much as possible, and I thought, 'Well, what's the fun in that?'... Eventually, I went to the admissions office, and I walked up to the counter with my little piece of paper and I said 'I want to change my enrolment' and they said 'No need. It's right. You start medicine in two weeks. See you later'. And I said 'No, no. I want to change my enrolment. I want to do mathematics.'

In 1981 Christine O'Keefe graduated from a Bachelor of Science and in 1982 a Bachelor of Science with First-Class Honours in Pure Mathematics. In 1985 she began her PhD *'Concerning t-spreads of $PG((s+1)(t+1)-1,q)$'.* in which she spent a year at the University of Rome under an Italian Government Exchange Scholarship.[5] To this day the University of Rome is a world leader in the study of Projective Finite Geometry, a research area in which Chrsitine O'Keefe would continue to publish at least until 2008. In the meantime O'Keefe would be a Lecturer then Research Fellow (1989-90) at the University of Perth before her return to the University of Adelaide in 1991. From 1995 to 2000 Christine O'Keefe would hold an ARC Queen Elizabeth II Fellowship and a Senior Research Fellowship.[5]

Of particular interest on a personal level is a series of papers published in the early 90's on Ovoids of $PG(3,q)$.

The space $PG(3,q)$ is a 3 dimensional space in which on every line their lies $q+1$ points, in every plane lies q^2+q+1 lines, and there are q^3+q^2+q+1 planes. Additionally, every pair of coplanar lines intersect at a point, and every pair of planes intersect in a line. There is a certain way to coordinatise $PG(3,q)$.

An *ovoid* is a set of q^2+1 points in $PG(3,q)$ such that no three points are collinear.

As of yet, we only know of two types of ovoid, those that are the solution to a second order homogeneous polynomial, known as the elliptic quadrics, and another type, the Suzuki-Tits ovoids. It is an ongoing research problem to see if for any q another type of ovoid exists in $PG(3,q)$. In 1962 the ovoids of $PG(3,8)$ were classified with computer aid. In 1990 O'Keefe authored a paper with Tim Penttila proving that all the ovoids of $PG(3,16)$ are elliptic quadrics via computer proof. O'Keefe and Penttila would likewise conduct a non-computer proof in 1992.[4]

About this time Christine O'Keefe would develop an interest in areas of information security in which her knowledge of Finite Geometry found particular use. Namely, that of secret-sharing schemes, in which a key must be shared amongst a number of users.

O'Keefe found she enjoyed doing the kind of work people would actually use. It wasn't long before she saw a job advert at the CSIRO, looking for a mathematician.[3]

> It's actually a funny story. The applications had closed and I met none of the selection criteria but I rang them to ask if they'd still accept an application, and they said yes, so I applied. I didn't get the job I applied for, but they made a new one for me... It was quite scary to leave academia, which was the only real job I'd ever had, and take a step outside that and change research area. That's a big thing, because suddenly everything I knew was irrelevant——all of the journals I knew weren't relevant anymore, along with my network of contacts. It was a challenge to start again in a new area. Probably the hardest time was during the first international conference on finite geometry after I'd left the field. I didn't go but all my buddies were there. I missed it then.

O'Keefe would continue to work at the CSIRO from 2000 onwards, returning to the University of Adelaide in 2011 as an Adjunct Professor. O'Keefe has a long list of achievements at the CSIRO and the University of Adelaide for her research. In particular in 2000 she was awarded the Australian Mathematical Society Medal, the first woman to win the medal in its 20 year history.[3]

> It was absolutely a delight; wonderful! I felt it was a great honour. I remember when I got the call—it was fabulous; quite a surprise... It was actually quite interesting, because the day that the medal was presented was the day after my 40th birthday—you know you have to be nominated before you're 40—and I had organised to have a birthday party on the Saturday night. I'd hired a hall and invited a hundred people. I thought that the presentation would be on the Monday, but the presentation was going to be in Brisbane at 10 o'clock on Sunday, the next morning! I did manage to get there. I'd had no sleep, having finished the party at about 3 am. I caught the plane to Brisbane at 5:30, walked in slightly late and then gave a talk about my work.

In 2018 O'Keefe retired from her work at both the University of Adelaide and the CSIRO.

Sam and the Aliens

Sam sat, and began reading the latest report. 'These aliens are *strange*,' thought Sam, slowly blinking in surprise—or at least the anthropomorphic equivalent of blinking. In reality, Sam momentarily recessed a pair tubular eyes into themselves. The type of tubular eyes which only occur in one known vertebrate, the Spookfish. Luckily, Sam was not a vertebrate.

The report Sam was reading detailed the findings of an archaeological survey. A team had discovered an abandoned alien probe. Comparison between the state of the probe's nuclear reactor and the environmental weathering of the probe's exterior revealed they were close to finding the alien homeworld—This probe existed for just 12 of Sam's celestial periods outside of its current environmental conditions. This put a very low upper limit on the interplanetary distance the probe may have traveled. A quick mental calculation told Sam to expect a report on the alien homeworld very soon indeed.

This was not, by itself, unexpected. Sam's 'people' have found many probes, so were not surprised by the discovery of the probe itself. Sam's people were able construct a probabilistic location of origin from these probes, so were well prepared to be closing in on the aliens. In fact, a mission was well underway to scout the most likely homeworld. What Sam found strange, as Sam did for almost every probe, was the density of the probe.

Not the density in the literal sense but rather the density of the sensors; the circuitry; the recievers and communicators; the telemetry and support systems; all the parts that gave the probe its functionality.

Sam lent back in their chair (Flattened themself into a disk), and reflected, for a moment. At present, of the 36 probes known to Sam, only 4 carried at least one module without obvious functionality. The first two carried a gold plated disk, with detailed, ornamental etchings. The next two carried gold plaques, with less intricate—but still clearly ornamental—etchings.

Sam, and other academics, were deeply concerned by this. Why would an alien race, clearly technologically capable of spacefaring, pack their probes so densely?

Some postulated that the aliens rather spent their resources on other pursuits—literature; multimodal art; architecture—and spared nothing for exploration. Others conjectured it was the pursuit of knowledge itself that these aliens valued. That they valued knowledge and exploration and had little to spare on embellishment. Or perhaps the aliens lived in a culture of extreme conservationism—where to create and expel an object from the ecosystem in which it is formed, never to return, is a great taboo, and only done sparingly.

The most popular theory, and what Sam believed, is that these aliens were dying.

That the alien homeworld was at a point of absolute scarcity. That the etchings on the first probes were there because that is all the aliens could afford. Sam believed, these probes were designed as efficiently as possible so as to find some way to save the aliens, while wasting as few resources as possible.

Sam believed, that their people could save these aliens.

Sam was brought back to the present by the quiet beep of a new report. Hyphae stood on end. Could this be it? the first observation of the alien homeworld?

The report was opened as quickly as inhumanely possible.

Sam was appalled.

The aliens did not live on a planet of scarcity. They lived on a great green-blue sphere of natural abundance.

It seemed that no theory about the alien civilisation survived reality.

Sam felt unwell, as if someone had put him together all wrong. The aliens *were* strange.

- George Savvoudis, ColAUMS Editor

References

[1] University examinations. *South Australian Register* (13 Dec 1884), 4. Available at https://trove.nla.gov.au/newspaper/page/4044378.

[2] Barrow-Green, J. *Poincaré and the three body problem*. American Mathematical Soc., 1997.

[3] O'Keefe, C. Mathematical minds. *The Australian Mathematical Society Gazette 2* (2010), 74–77.

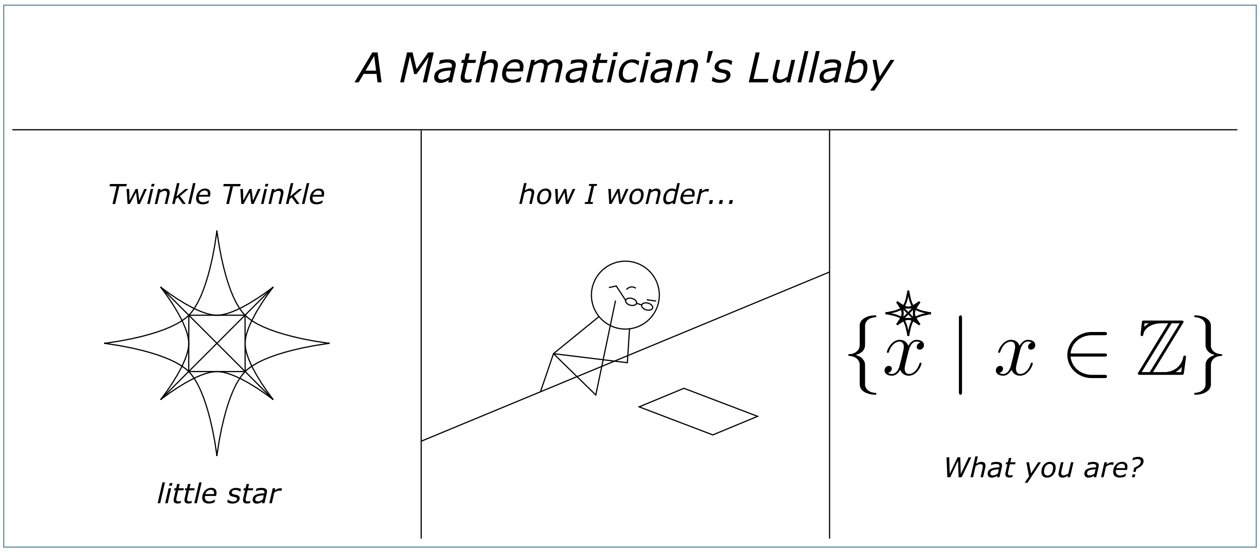
[4] O'Keefe, C., and Penttila, T. Ovoids of $PG(3, 16)$ are elliptic quadrics, ii. *J Geom 44* (1992), 140–159.

[5] Ward, C. https://csiropedia.csiro.au/okeefe-christine/, 2011.

Puzzle 2:

One day a multi-billionaire decided to build a floating island in the Pacific Ocean as far away from Mount Everest as possible (they hate mountains). In response to a recent Twitter post, a group of climbers decided it would be funny to place a mold of the billionaire's head atop Mount Everest.
As a show of force, the billionaire is building a massive cannon their floating island to shoot the mold off it's perch. However, a design flaw means the cannon can only point tangentially to the surface of the earth.
How fast does the cannonball need to be travelling to hit the mold. Assume no air resistance and the cannonball is of insignificant mass relative to the earth.

Hint: You may assume that the cannonball takes an elliptic path with one foci at the centre of the earth



ColAUMS Space is published by the Adelaide University Mathematics Society (AUMS).
The opinions and calculations contained herein are not necessarily those of AUMS or the editors.
Reasonable care is taken to ensure that all information is correct at the time of publication, however, neither AUMS nor *ColAUMS Space* take responsibility for any possible errors.
Because we're irresponsible.
This includes divisions by zero, logical fallacies and wild assumptions.
Contact *ColAUMS Space*:
colaums.space@gmail.com
Edited by George Savvoudis.