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More articles

Can't get enough maths? Visit aums.org.au/colaums for articles on maths, history, profiles, puzzles and more!

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Contribute

Have an idea for an article or want to contribute? Visit the website or contact the editor at colaums.space@gmail.com

Writing Competition!

This semester, AUMS is running a competition to encourage more people to contribute to ColAUMS Space! The winner will receive a shiny \$50 Dymocks gift voucher. To enter, simply write an article to appear in print or on the website, write a historical profile, or submit a puzzle. For rules and more information visit aums.org.au/colaums/contribute-to-colaums/competition/. Prizes will be drawn Monday in week 12!

- Alex Schutz, ColAUMS Space Editor

Smallest Sudoku

The Sudoku has been a puzzling staple since its introduction in 1979. Most newspaper Sudokus will have around 25 *clues* - pre-filled numbers - with more added as the difficulty decreases. But if you have ever tried to generate a Sudoku on your own, you may have wondered - what is the smallest number of clues for the sudoku to be uniquely solvable?

			8	1				
							4	3
5								
			7		8			
					1			
	2		3					
6							7	5
		3	4					
			2		6			

The smallest number of clues for a sudoku is 17

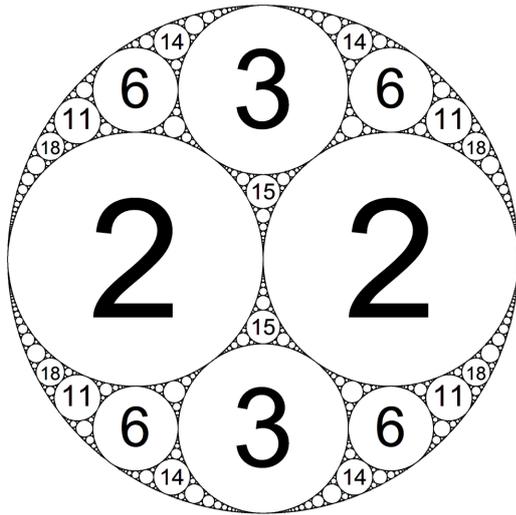
In 2012, Gary McGuire of University College Dublin showed that the minimum number of clues necessary to completely define a Sudoku is 17. A puzzle with 16 or fewer solutions will never have a unique solution. Of course, having 17 clues does not guarantee a unique solution.

Currently, the most clues for a *minimal* Sudoku is believed to be 40, of which only two are known. If any clue is removed from either of these Sudokus, the puzzle would have more than one solution.

As for the most clues possible in a Sudoku while still not rendering a unique solution, it is four short of a full grid (77).

Apollonian Gasket

From t-shirts to Buddhist temples, the Apollonian Gasket is an aesthetic wonder created from the simplest of shapes. To create one, simply draw three mutually tangent circles of any size. It is generally possible to then find 2 circles which are tangent to all three of these initial circles - one in the centre of them and one encompassing them. From there, the gasket is iteratively created by drawing the circle tangent to any three of the circles already drawn.



An Apollonian Gasket with integer values

But, given three circles, how do you draw a fourth circle that is exactly tangent to all three? The first mathematician to seriously consider this question was Apollonius of Perga, a Greek geometer who lived in the third century B.C. It wasn't until the 17th century, however, until René Descartes found (but incorrectly proved) a formula connecting the radii of the four circles. If a circle has radius r , we define its curvature to be $1/r$. For four circles of curvature a , b , c and d , (note that for a circle containing the other circles, the curvature is negative) Descartes' formula is:

$$a^2 + b^2 + c^2 + d^2 = \frac{(a + b + c + d)^2}{2}$$



Historical Profile: Euclid

Described as the "Father of Geometry", Euclid's mathematical influence can be felt to this day. Despite the longevity of his work, very little is known about his life. Euclid was born around 365 B.C. in Alexandria, Egypt and lived until about 300 B.C. Euclid's most famous work is his collection of 13 books, dealing with geometry, called *The Elements*. In Book I, he begins with 5 postulates (including the famous parallel postulate which is key to generating 'Euclidean' geometries), and sets off to derive fundamental properties of geometry based upon these axioms. These axiomatic and constructive methods were widely influential, and served to revolutionise the idea of mathematical proof into what it is today.

In fact, the very same equation can be used to compute the location of the fourth circle as well, and thus completely solve the drawing problem. This fact was discovered in the late 1990s by Allan Wilks and Colin Mallows of AT&T Labs.

In 1936, Nobel laureate (in chemistry) Frederick Soddy became mesmerised by the charm of Apollonian Gaskets. He published in *Nature* a poetic version of Descartes' theorem, which he called "*The Kiss Precise*":

*Four circles to the kissing come
The smaller are the benter.
The bend is just the inverse of
The distance from the center.
Though their intrigue left Euclid dumb,
There's now no need for rule of thumb.
Since zero bends a dead straight line,
And concave bends have minus sign,
The sum of the squares of all four bends
Is half the square of their sum.*

Since this prosodic effort, it has become tradition to publish any extension of his theorem in poetic form as well. Clearly mathematics and art are linked in more ways than one.

Puzzle: Paper Folding

Divide a piece of paper into 8 sections and write numbers on it according to one of the pictures. Fold the paper along the lines so that the numbers are sorted (number 1 on top, 2 under it, etc.).

1	8	7	4
2	3	6	5

1	8	2	7
4	5	3	6