ColAUMS Space
ADELAIDE UNIVERSITY MATHEMATICS SOCIETY

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In this issue

Upcoming events

BBQ	-	29/05
Quiz Night	-	06/06

More articles

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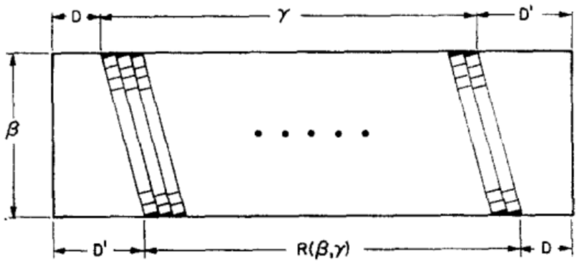
Packing a Square

It's another long day at the Rubik's cube factory, and the packing machine has broken down again. As you slowly pack the cubes 1-high in an enormous square tray, you wonder about way the cubes are packed. The tray can fit around 47145.01 cubes along its side (I told you it was huge). Now, obviously the extra 0.01 cubes is nearly insignificant, but as you pick up cubes and pack them in you think about the wasted area. If you pack the cubes in a 47145×47145 formation, the leftover area is about 942.9 squares worth - think of all the extra cubes you could pack! But how might this space be filled? Surely packing the cubes on a skewed angle would be counter-productive.

In 1975, Paul Erdős and R. L. Graham worked out that in fact for large side lengths α, it is useful to pack the squares askew. You begin by packing the corner in the normal configuration with N^2 blocks, where $\lfloor N = \alpha - \alpha^{8/11} \rfloor$. The remaining space can be decomposed into two rectangles of width $\beta = \alpha - N$. These rectangles can now be packed with rows of squares at a tilt, with $\lfloor \beta \rfloor + 1$ squares in each row (spanning the width), tilted at an angle so that the top and bottom corners of each row touch the sides. The remaining trapezoids can be filled in a similar way with rows of tilted squares just slightly longer than what can be packed straight. This packing configuration will result in an uncovered area of $\alpha^{7/11}$.



Erdős' square packing method

What does this mean for your Rubik's cube packing? Well, after you meticulously follow the method, packing the cubes at *just* the right angles, you eventually manage to fit... exactly one extra Rubik's cube in the tray. Excited about your new-found source of efficiency, you run to tell your boss - who promptly tells you to stop faffing around with all of this maths nonsense and get back to work. Maybe your efforts would be better appreciated in the applied maths department of a university.

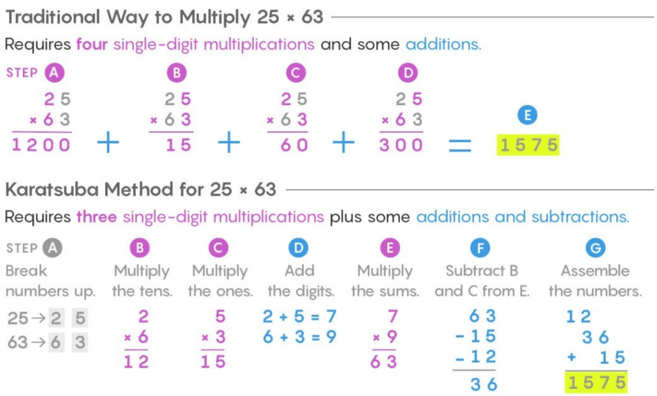
To read more about this solution, see Erdős, P. and Graham, R. L. (1975), doi: 10.1016/0097-3165(75)90099-0. In fact, the wasted space has since been reduced to $\alpha^{3/5}$.

The Fastest Way to Multiply

On the 18th of March, multiplication was revolutionised when two researchers found the fastest method yet for multiplying large numbers.

The primary school or "carrying" method or multiplication that we all learned requires about n^2 steps, where n is the number of digits of each of the numbers being multiplied. This works well for numbers with just a few digits, but it bogs down for numbers with millions or billions of digits (which is needed to accurately calculate π or as part of the worldwide search for large primes).

For millennia it was widely assumed that there was no faster way to multiply. Then in 1960, the 23-year-old Russian mathematician Anatoly Karatsuba found a method which involves breaking up the digits of a number and recombining them in a novel way that allows you to substitute a small number of additions and subtractions for a large number of multiplications, resulting in only $2n$ steps.



Multiplication methods

When dealing with large numbers, the Karatsuba procedure can be repeated, splitting the original number into almost as many parts as it has digits. With each splitting, multiplications that require many steps to compute are replaced with additions and subtractions that require far fewer.

Karatsuba's method made it possible to multiply numbers using only $n^1.58$ single-digit multiplications. Then in 1971 Arnold Schönhage and Volker Strassen published a method capable of multiplying large numbers in $n \log n \log(\log n)$ multiplicative steps. For two 1-billion-digit numbers, Karatsuba's method would require about 165 trillion additional steps.

In that same paper Schönhage and Strassen conjectured that there should be an even faster algorithm than the one they found a method that needs only $n \log n$ single-digit operations and that such an algorithm would be the fastest possible.

Last month, David Harvey and Joris van der Hoeven got there. Their method is a refinement of the major work that came before them. It splits up digits, uses an improved version of the fast Fourier transform, and takes advantage of other advances made over the past forty years. Harvey and van der Hoeven's algorithm proves that multiplication can be done in $n \log n$ steps. However, it does not prove that there is no faster way to do it, which remains an open problem.

Puzzle: Fork in the Road

Local logician David Butler is visiting the South Seas, and has found himself at a fork, wanting to know which of two roads leads to the village. Present are three willing natives, one each from a tribe of invariable truth-tellers, a tribe of invariable liars, and a tribe of random answerers. Of course the David doesn't know which native is from which tribe. Moreover, he is permitted to ask only two yes-or-no questions, each question being directed to just one native. Can he get the information he needs?

Historical Profile: Hilbert

David Hilbert was a German mathematician whose name and contributions crop up in many areas, from the Hilbert Hotel to his 23 Hilbert Problems. Hilbert was born in Königsberg, a town already steeped in mathematical history. Hilbert accepted a professorship at the University of Göttingen in 1895 where he added to the university's continuing fame. Around 1925, Hilbert developed pernicious anemia, a then-untreatable vitamin deficiency whose primary symptom is exhaustion; his assistant said he "was hardly a scientist after 1925, and certainly not a Hilbert". By the time Hilbert died in 1943, the Nazis had nearly completely restaffed the university, as many of the former faculty had either been Jewish or married to Jews. News of his death only became known to the wider world six months after he died.